Routine and non-routine problem solving

We can categorize problem solving into two basic types: routine and non-routine. The purposes and the strategies used for solving problems are different for each type.

Routine problem solving

From the curricular point of view, routine problem solving involves using at least one of the four arithmetic operations and/or ratio to solve problems that are practical in nature. Routine problem solving concerns to a large degree the kind of problem solving that serves a socially useful function that has immediate and future payoff. Children typically do routine problem solving as early as age 5 or 6. They combine and separate things such as toys during their normal activities. Adults are regularly called upon to do simple and complex routine problem solving. Here is an example.

> *A sales promotion in a store advertises a jacket regularly priced at $125.98 but now selling for 20% off the regular price. The store also waives the tax. You have $100 in your pocket (or $100 left in your charge account). Do you have enough money to buy the jacket?*

As adults, and as children, we normally want to solve certain kinds of problems (such as the one above) in a way that reflects an '*Aha, I know what is going on here and this is what I need to do to figure out the answer.*' reaction to the problem. We do not want to guess and check or think backwards or make use of similar strategies. Invariably, solving such problems involves using at least one of the four arithmetic operations (and/or ratio). Being good at doing arithmetic (e. g. adding two numbers: mentally, by pencil and paper, with manipulatives, by punching numbers in a calculator) does not guarantee success at solving routine problems. <u>The critical matter is knowing **what** arithmetic to do.</u> Doing the arithmetic is secondary to the matter.

A mathematics researcher interviewed children about how they solve routine problems. One boy reported his method as follows: *If there were two numbers and they were both big, he subtracted. If there was one large and one small number, he divided. If it did not come out even, he multiplied.* The other interesting aspect of all of this is that the child had done quite well at solving routine problems throughout his school career. What does this say about teaching practice? What does this say about assessing what children understand?

Is the case of the boy an isolated incident or is it the norm? Unfortunately, research tells us that it is likely the norm. Not enough students and adults are good at solving routine problems. Research also tells us that in order for students to be good at routine problem solving they need to learn the meanings of the arithmetic operations (and ratio) well and in ways that are based on real and familiar experiences. While there are only four arithmetic operations, there are more than four distinct meanings that can be attached to the operations. For example, division has only one meaning: splitting up into equal groups. Subtraction, on the other hand, has at least two meanings: taking away something away from one set or comparing two sets.

Once students understand the meaning of an arithmetic operation they have a powerful conceptual tool to apply to solving routine problems. The primary strategy becomes deciding on what arithmetic operation to use. That decision cannot be made in the manner done by the boy of the research anecdote. The decision should be made by IDENTIFYING WHAT IS GOING ON IN THE PROBLEM. This approach requires understanding the meanings of the arithmetic operations.

The research evidence suggests that good routine problem solvers have a repertoire of automatic symbol-based and context-based responses to problem situations. They do not rely on manipulating concrete materials, nor on using strategies such as 'guess and check' or 'think backwards'. Rather, they rely on representing what is going on in a problem by selecting from a limited set of mathematical templates or models.

Solving routine problems should at some point involve solving complex problems. Complexity can be achieved through multi-step problems (making use of more than one arithmetic operation) or through Fermi problems. It is advisable to do both.

Fermi problems are special problems that are characterized by the need to estimate something and the need to obtain relevant data. They typically involve the application of the meaning of at least one arithmetic operation and sometimes something else (e. g. how to calculate the area of a triangle). Here is an example of a Fermi problem: *How many cars are there in Manitoba?* Solving this Fermi problem about the cars would involve matters like obtaining/estimating data about the population of Manitoba that might own a car and making use of the 'groups of' meaning of multiplication. It could involve more matters. That would depend on the degree of sophistication of insight into the problem.

In general, solving Fermi problems involves estimating where the exact value is often unknown, and perhaps it is even unknowable. While the estimate may be considerably in error, the important matter is on describing how the estimate was obtained. That requires students to justify their reasoning in terms of the meanings of arithmetic operations and in terms of the relevance of the data they collected/estimated.

Non-routine problem solving

Non-routine problem solving serves a different purpose than routine problem solving. While routine problem solving concerns solving problems that are useful for daily living (in the present or in the future), non-routine problem solving concerns that only indirectly. Non-routine problem solving is mostly concerned with developing students' mathematical reasoning power and fostering the understanding that mathematics is a creative endeavour. From the point of view of students, non-routine problem solving can be challenging and interesting. From the point of view of planning classroom instruction, teachers can use non-routine problem solving to introduce ideas (SET SCENCE stage of teaching); to deepen and extend understandings of algorithms, skills, and concepts (MAINTAIN stage of teaching); and to motivate and challenge students.

There are other uses as well. Having students do non-routine problem solving can encourage the move from specific to general thinking; in other words, encourage the ability to think in more abstract ways. From the point of view of students growing to adulthood, that ability is becoming more important in today's technological, complex, and demanding world.

Non-routine problem solving can be seen as evoking an '*I tried this and I tried that, and eureka, I finally figured it out*.' reaction. That involves a search for heuristics (strategies seeking to discover). There is no convenient model or solution path that is readily available to apply to solving a problem. That is in sharp contrast to routine problem solving where there are readily identifiable models (the meanings of the arithmetic operations and the associated templates) to apply to problem situations.

The following is an example of a problem that concerns non-routine problem solving.

> *Consider what happens when 35 is multiplied by 41. The result is 1435. Notice that all four digits of the two multipliers reappear in the product of 1435 (but they are rearranged). One could call numbers such as 35 and 41 as pairs of stubborn numbers because their digits reappear in the product when the two numbers are multiplied together. Find as many pairs of 2-digit stubborn numbers as you can. There are 6 pairs in all (not including 35 & 41).*

Solving problems like the one above normally requires a search for a strategy that seeks to discover a solution (a heuristic). There are many strategies that can be used for solving unfamiliar or unusual problems. The strategies suggested below are teachable to the extent that teachers can encourage and help students to identify, to understand, and to use them. However, non-routine problem solving cannot be approached in an automatized way as can routine problem solving. To say that another way, we cannot find nice, tidy methods of solution for all problems. Inevitably, we will be confronted with a situation that evokes the response; "*I haven't got much of a clue how to do this; let me see what I can try.*"

The list below does not contain strategies like: 'read the question carefully', 'draw a diagram', or 'make a table'. Those kinds of strategies are not the essence of what it takes to be successful at non-routine problem solving. They are only preliminary steps that help in getting organized. The hard part still remains - to solve the problem - and that takes more powerful strategies than drawing a diagram, reading the question carefully, or making a table. The following list of strategies is appropriate for Early and Middle Years students in that the strategies involve ways of thinking that are likely to be comfortable for these students.

* Look for a pattern
* Guess and check
* Make and solve a simpler problem.
* Work backwards.
* Act it out/make a model.
* Break up the problem into smaller ones and try to solve these first.

It is important that students share how they solved problems so that their classmates are exposed to a variety of strategies as well as the idea that there may be more than one way to reach a solution. It is unwise to force students to use one particular strategy for two important reasons. First, more than one strategy can usually be used to solve a problem. Second, the goal is for students to search for and apply useful strategies, not to train students to make use of a particular strategy.

Finally, non-routine problem solving should not be reserved for special students such as those who finish the regular work early. All students should participate in and be encouraged to succeed at non-routine problem solving. All students can benefit from the kinds of thinking that is involved in non-routine problem solving.

## Comparing routine and non-routine problem solving

To make clearer the distinction between routine and non-routine problem solving, consider the following two problems. Both are suitable for grade 3.

Problem 1

> *My mom gave me 35 cents. My father gave me 45 cents. My grandmother gave me 85 cents. How many cents do I have now?*

Problem 2

> *Place the numbers 1 to 9, one in each circle so that the sum of the four numbers along any of the three sides of the triangle is 20. There are 9 circles and 9 numbers to place in the circles. Each circle must have a different number in it.*
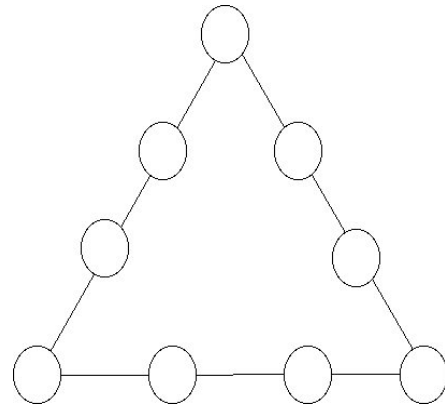
Notice that addition is required for both problems. In problem 1, **you need to figure out that you need to add**. Understanding addition as modeling a 'put together' action helps you realize that.

In problem 2, **you are told to add by the word 'sum'**. Understanding addition as modeling a 'put together' action does not help you with solving problem 2. Being good at arithmetic might help you a bit, but the matter really concerns a search for strategies to apply to the problem. Guess and check is a useful strategy to begin with.